

Challenges in visualizing results from more modern finite element formulations: high-order, DG, $H(\text{div})$, $H(\text{curl})$...

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Julien Fausty, Charles Gueunet, Mathieu Westphal and François Mazen

Kitware / Leader in AI & scientific open source solutions

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20+ years of expertise

Kitware USA, 1998
Kitware Europe, 2010



Revenue 2020

\$39M consolidated



Kitware / Services



TRAINING



SUPPORT

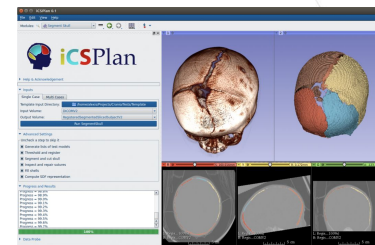
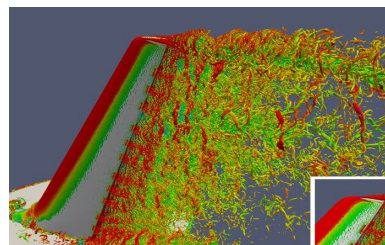
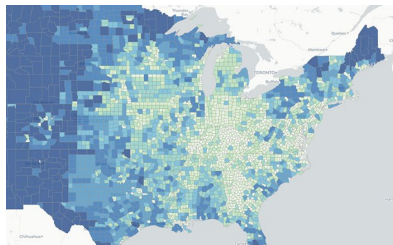
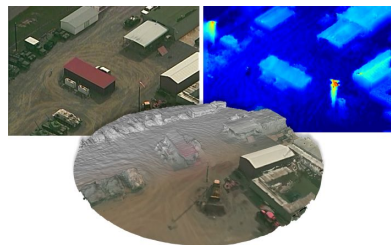


DEVELOPMENT



**GRANT
COLLABORATION**

Areas of expertise / Built on open source



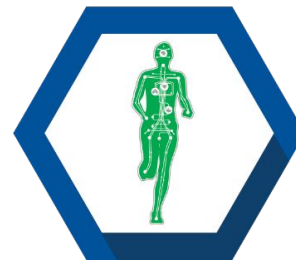
Computer
Vision



Data and
Analytics



Scientific
Computing

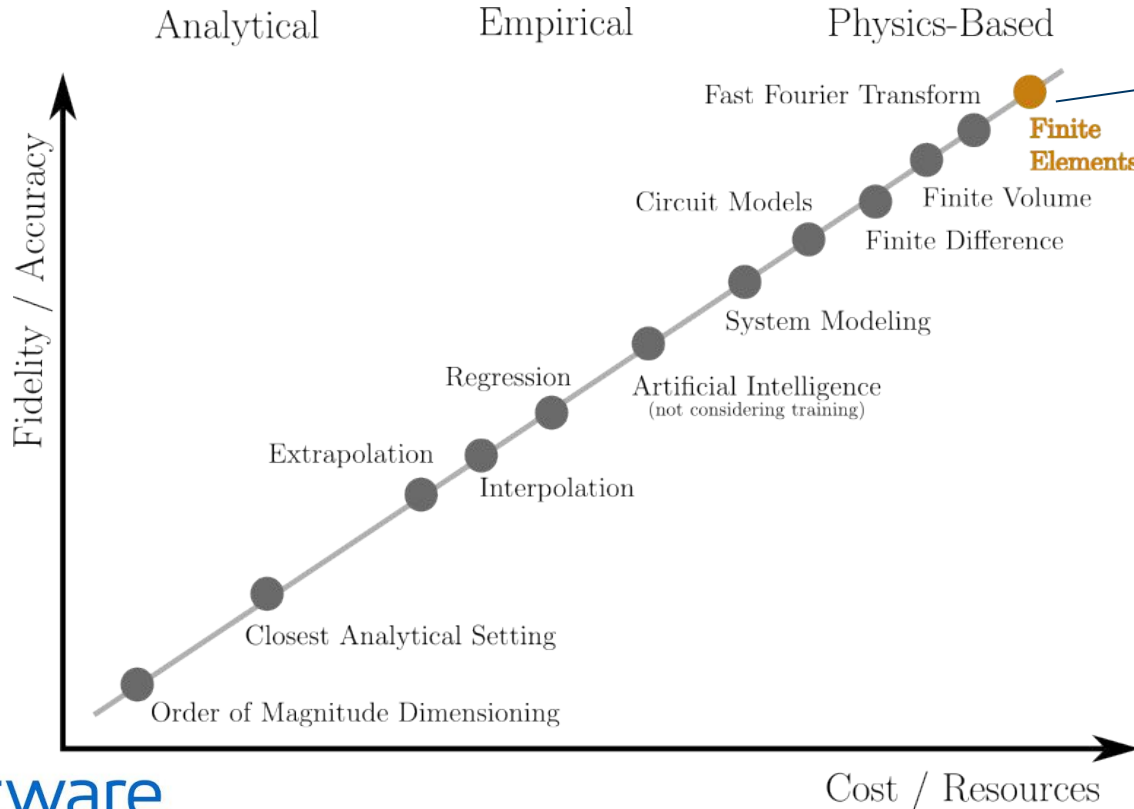


Medical
Computing



Software
Solutions

Numerical Methods: a cost/accuracy balancing act



A whole lot of other things that can affect this balance:

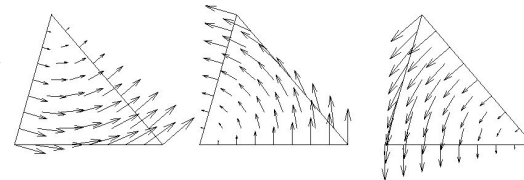
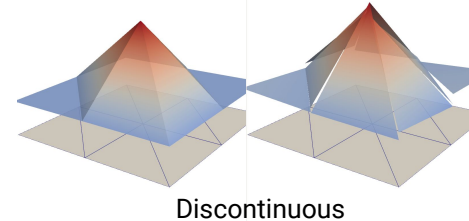
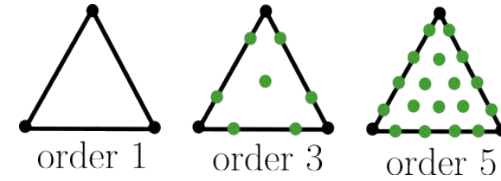
- Stability considerations
- Model choice
- Numerical schema
- Linear system solver
- Mesh resolution
- ...

What do we mean by "modern" finite elements?

Anything that is **not classical** finite elements.

Classical finite elements methods usually have the following properties:

- Piecewise linear per element solutions
- At least C^0 Continuity between elements
- Nodal degrees of freedom
- Scalar basis functions



Conforming

Jacobsson, Per. "Nedelec elements for computational electromagnetics." (2007).

Reasons for using more complex formulations

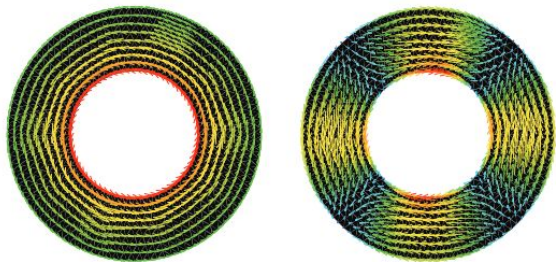


FIGURE 2.4. Approximation of the vector Laplacian on an annulus. The true solution shown here on the right is an (accurate) approximation by a mixed method. It is orthogonal to the harmonic fields and satisfies the differential equation only modulo harmonic fields. The standard Galerkin solution using continuous piecewise linear vector fields, shown on the left, is totally different.

Arnold, Douglas, Richard Falk, and Ragnar Winther. "Finite element exterior calculus: from Hodge theory to numerical stability." *Bulletin of the American mathematical society* 47.2 (2010): 281-354.

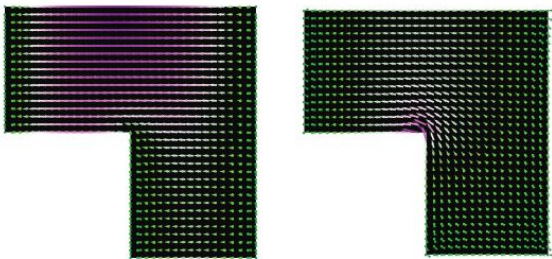
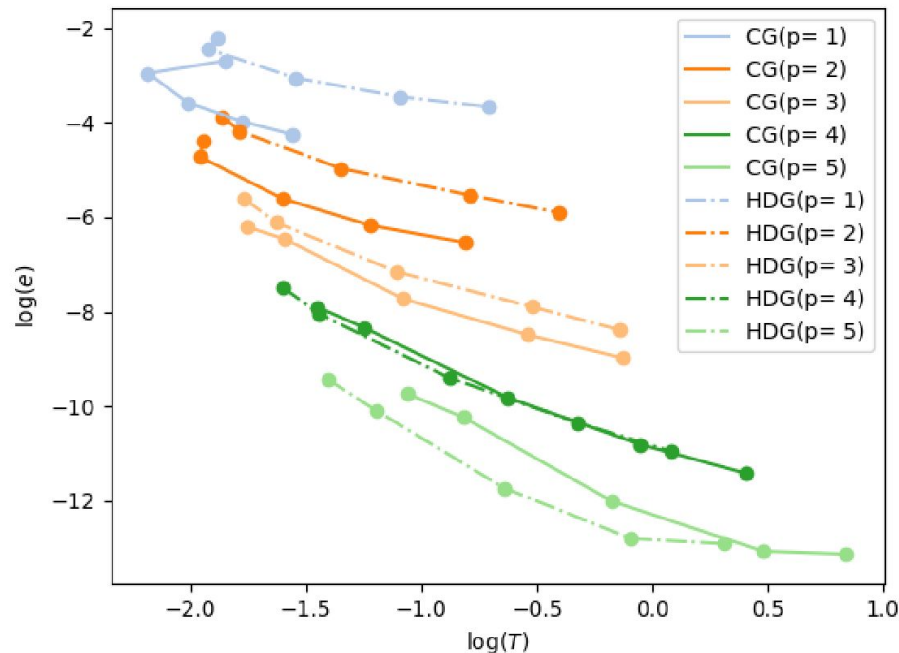
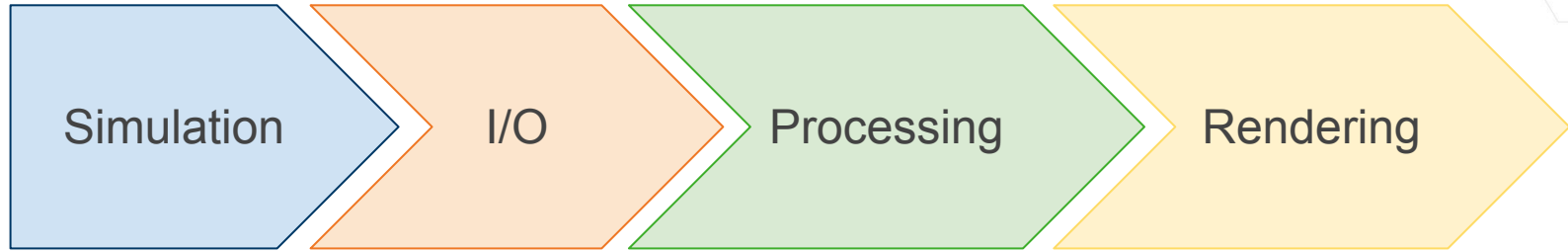


FIGURE 2.3. Approximation of the vector Laplacian by the standard finite element method (left) and a mixed finite element method (right). The former method totally misses the singular behavior of the solution near the reentrant corner.

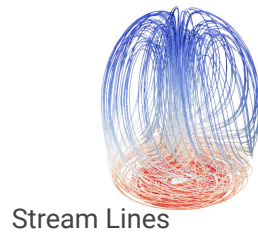
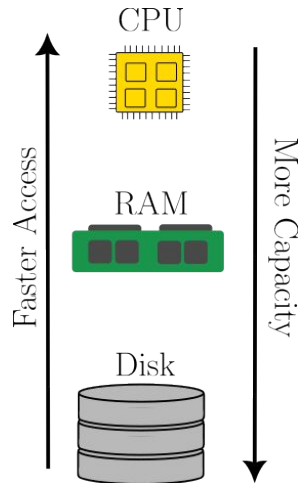
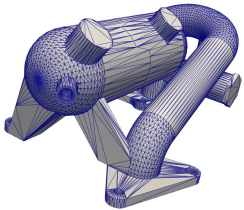


Logarithmic evolution of the error as a function of computational time for an analytical parabolic test case: a comparison between Continuous Galerkin (CG) and Hybrid Discontinuous Galerkin (HDG) approaches at high orders

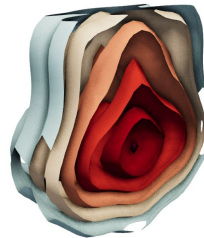
The big steps from simulation to visualization



$$\begin{cases} \int_{\Omega} F(u, \phi) d\Omega = \int_{\partial\Omega} b(\phi) d\Omega \\ \int_{\partial\Omega} B(u, \phi) d\Omega = \int_{\partial\Omega} f \cdot \phi d\Omega \end{cases}$$



Stream Lines



Contouring



The OpenFoam Motorbike, rendered with the NVIDIA OptiX path tracer in ParaView.

Formalizing the Challenges

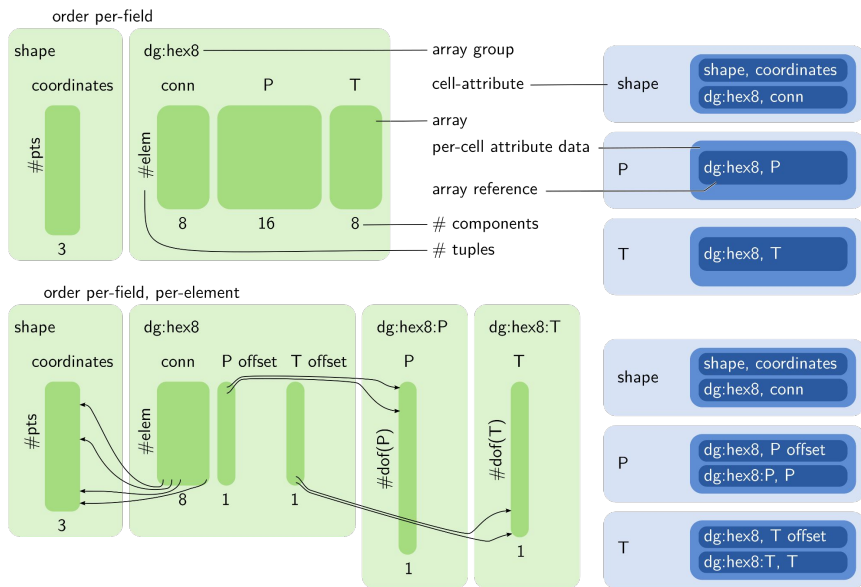
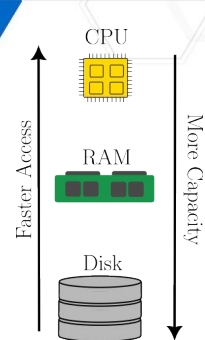


I/O: formatting and interfacing

What format can solutions from modern finite element codes use to describe themselves?

Characteristics of solutions from more modern formulations breaking the mould:

- A zoology of element types
 - Modal vs. nodal degrees of freedom
 - Non-standard interpolation primitives
- Discontinuous point data

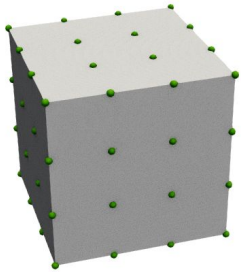


Design for discontinuous galerkin data structure taken from:

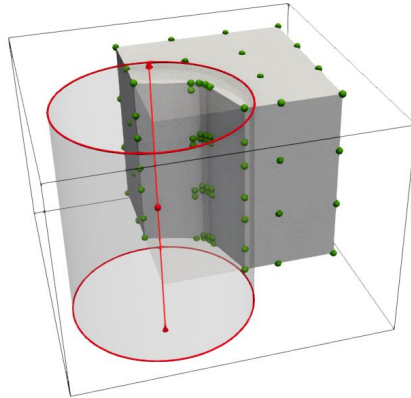
<https://discourse.vtk.org/t/discontinuous-galerkin-elements-and-other-novel-cell-types-function-spaces/9209>

Post-processing: reducing and transforming data

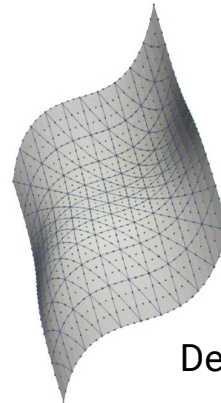
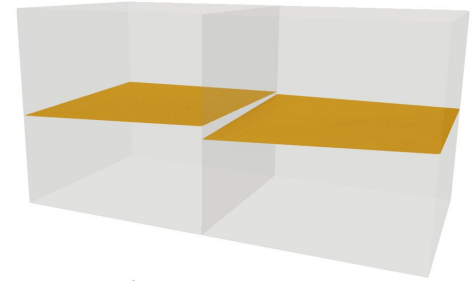
Do basic processing operations have the same implementations for more complex data?



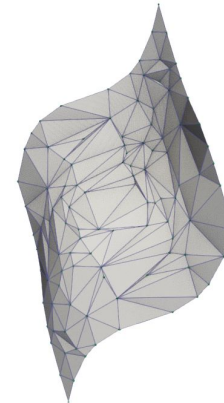
Clipping



Contouring



Decimation



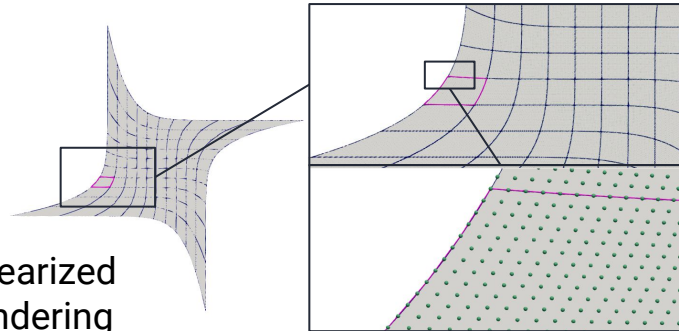
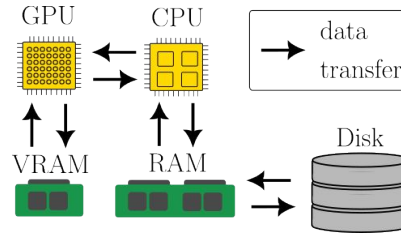
Rendering: from data to image



How do we implement graphics primitives based on user formulations? How should graphics primitives behave in discontinuous settings?

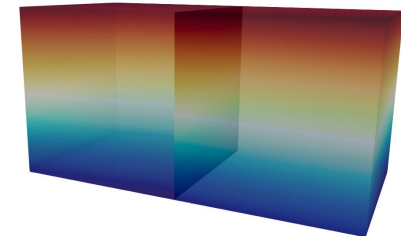
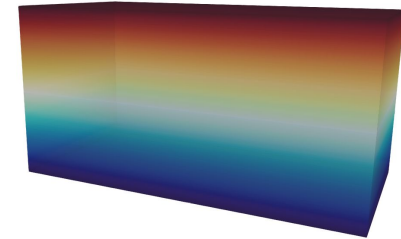
2 main challenges:

- Translating special interpolation primitives to the GPU
- Rendering discontinuous data



Linearized
Rendering

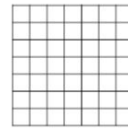
Discontinuous
Rendering
Artifacts



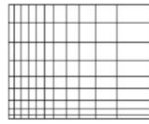
Infrastructure: refactoring tools for new applications

How do we migrate existing visualization tools towards supporting these new data models?

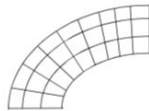
These more modern finite element formulations necessitate fundamental design changes in existing APIs, formats and computational kernels.



(a) Image Data



(b) Rectilinear Grid



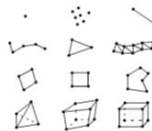
(c) Structured Grid



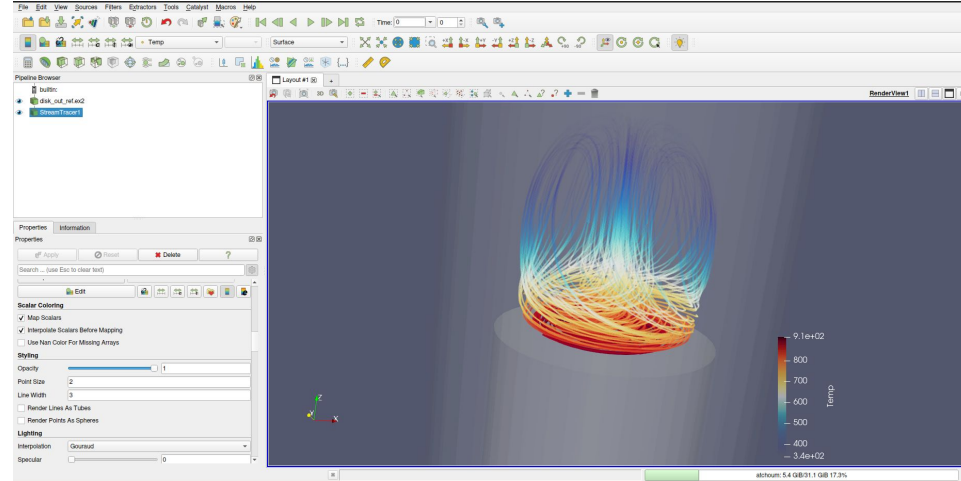
(d) Unstructured Points



(e) Polygonal Data



(f) Unstructured Grid



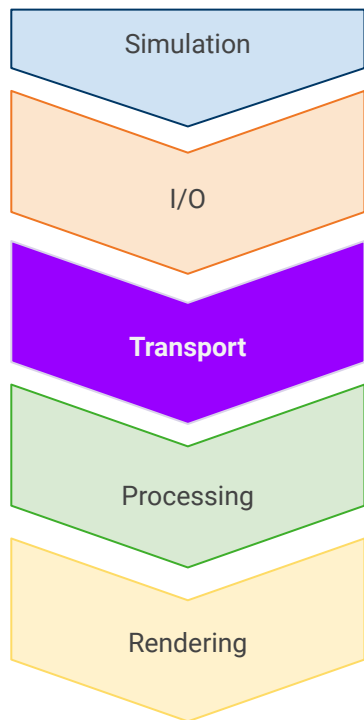
ParaView Screenshot

VTK Dataset model taken from:
<https://examples.vtk.org/site/VTBook/05Chapter5/>

Promising Solutions



Intermediate solution framework



Extra step to project the high-order / exotic data onto a linear mesh dedicated to visualization

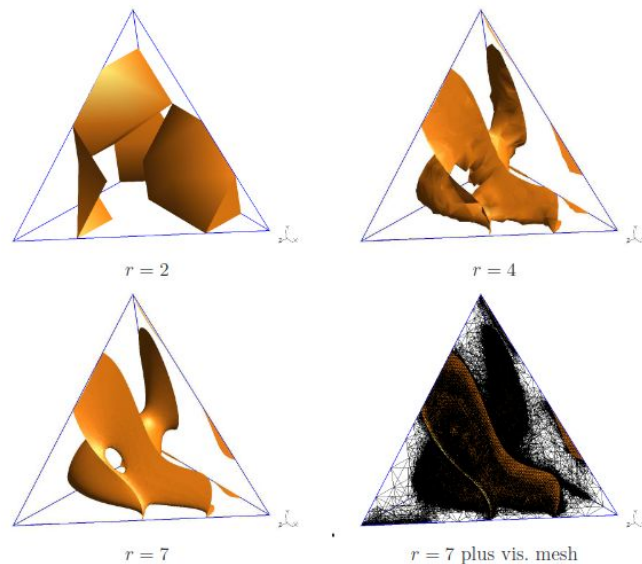


Figure 5. Visualization of one 4th order function on one single tetrahedron. Figures show one iso-surface using different error thresholds. The last Figure (bottom-right) show the visualization mesh together with the iso-surface.

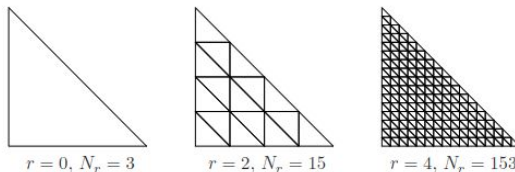
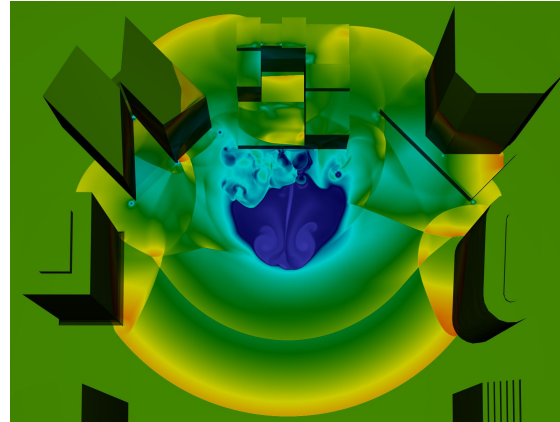


Figure 3. Some AMR refinement templates for triangles at different resolution levels.

J.-F. Remacle, N. Chevaugne, E. Marchandise, and C. Geuzaine. Efficient visualization of high-order finite elements. *International Journal for Numerical Methods in Engineering*, 69(4):750–771, 2007.

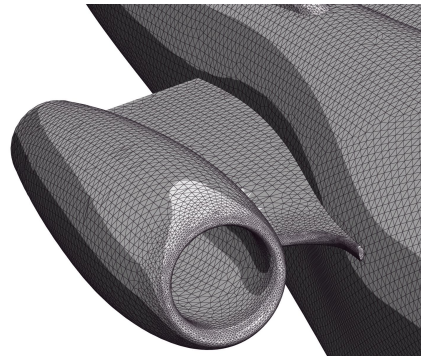
Long-term solution framework

Pixel exact approaches using appropriate computational primitives on the GPU.



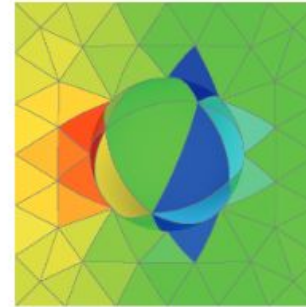
ViZiR: https://pyamq.saclay.inria.fr/images/vizir/city_blast_sol.png

Loseille, Adrien & Feuillet, Rémi. (2018).
Vizir: High-order mesh and solution
visualization using OpenGL 4.0 graphic
pipeline. 10.2514/6.2018-1174.



ViZiR: https://pyamq.saclay.inria.fr/images/vizir/falconP3_zoom.png

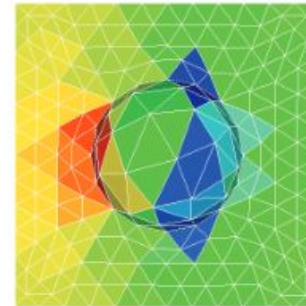
B. Nelson, E. Liu, R. M. Kirby, and R. Haimes.
Elvis: A system for the accurate and
interactive visualization of high-order finite
element solutions. IEEE transactions on
visualization and computer graphics,
18(12):2325–2334, 2012.



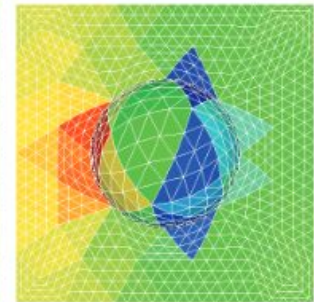
(a) EIVis



(b) Visual3 0 Refinements



(c) Visual3 1 Refinement



(d) Visual3 2 Refinements

Fig. 7. A top-down view of the hemisphere from Case 3. The mesh plotting tools of Visual3 and EIVis are enabled.

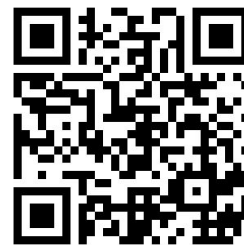
Conclusion and Outlook

Conclusions:

- Modern finite element formulations are becoming more mainstream (with good reason)
- Challenges in visualizing these kind of datasets can be divided into 4 categories:
 - Absence of existing standard for **I / O** formatting and design
 - Classical **post-processing** transformations do not behave correctly for more complex data
 - **Rendering** primitives need to be re-designed for these types of datasets
 - Porting current **infrastructure** is not trivial
- 2 generic approaches to these problems stand out: linear projections and native support

Outlook:

- A clear and iterative roadmap needs to be elaborated at the community level
- Best to source dedicated project financing for affecting coherent change



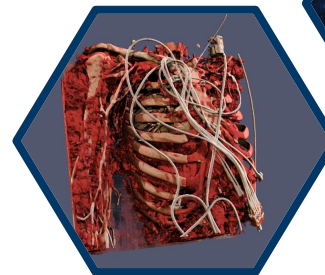
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- Demo space
- ParaView dinner (optional)

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Thank you for your attention!

Questions?

Or/and contact us at kitware@kitware.fr or julien.fausty@kitware.com