Topological Data Analysis for numerical method comparisons of 2D turbulent flows

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HYPERION ingredients
HYPERsonic vehicle design with Immersed bOuNdaries

\[ \frac{d}{dt} \int_{\Omega} \mathbf{u}(\mathbf{x}, t) d\mathbf{x} = \int_{\partial \Omega} \mathbf{F}(\mathbf{u}(\mathbf{x}, t)) \mathbf{n} ds \]

Finite Volume
Cartesian Structured Mesh

Direct Numerical Simulation
Large Eddy Simulation

HYPERION

Sharp Immersed Boundaries

Parallel I/O

Finite Volume
Cartesian Structured Mesh

Direct Numerical Simulation
Large Eddy Simulation

Parallel I/O
**Kelvin-Helmholtz Instability**

- Pressure \((p)\), velocity \((u,v)\) and density \((\rho)\)

- Mesh size: 512*512 on Cartesian grid

- Boundary conditions: periodic

**Dataset**

- Input dataset: simplicial complexes, linear scalar field

- Common measure for turbulent flow, enstrophy: \( \mathcal{E} = 0.5 \left| \nabla \times \mathbf{u} \right|^2 \)
Goal

Find the best numerical method to reduce the global time of the simulation and help scientists to choose the best numerical method to describe 2D turbulent flow.
Why to move to TDA ?

Complex and large dataset

► Comparison between many different cases
► Comparison between different features: scales, vortices, recirculation bubble
► A lot of noise and perturbation with turbulent flow, need to use filter

Why use topological data analysis

► Identify vortex centers \(\rightarrow\) Extract critical points
► Visual representation of the enstrophy maxima (critical points) \(\rightarrow\) Persistent diagrams
► Noise removal of the enstrophy variable \(\rightarrow\) Persistence threshold
► Comparison of all simulation runs \(\rightarrow\) Wasserstein distance
Open-source TDA library

► ~120k lines in C++, BSD license
► Python bindings, binary packages
► http://topology-tool-kit.github.io

TTK provides

► Topological tools require to extract complex features
► End user analysis tool integrated into Paraview
► Great for interdisciplinary research!
Introduction to Persistence Homology : Persistence

Persistence

- Abstraction : Order topological features in term of importance or noise
- Evolution of the topology of sublevel sets
- Topological features are created (and destroyed) at critical points
- The lifetime of a topological feature is called "Persistence"
Periodic condition

Non Periodic conditions

Periodic conditions
Use of the Persistence Homology: Filtering

No Filter

Filter
**Wasserstein distance**

- Distance between distributions known as the « earth mover’s distance »
- Optimal transport problem: Minimal cost of moving one persistence diagram to the other
Use of the Persistence Homology: Wasserstein distance
Matrix distance between numerical methods

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>WENOZ-5-HLL</td>
<td>W5H</td>
</tr>
<tr>
<td>WENOZ-5-AUSMUP</td>
<td>W5A</td>
</tr>
<tr>
<td>WENOZ-7-HLL</td>
<td>W7H</td>
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<td>WENOZ-7-AUSMUP</td>
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</table>

This diagram illustrates the comparison of different numerical methods, showing the matrix distances between them. The colors range from red (CLOSE) to green (FAR), indicating the level of similarity or difference.
Conclusion and Future work

Lessons learn thanks to TDA

► Confirm the independance of the orders
► Ease the identification of the scheme/order/solver

More to come ...

► Apply this method at a larger scale (hundred of cases and runs)
  ◆ More solvers
  ◆ Different level of turbulence
  ◆ Different mesh resolutions
► Continue with TDA for vortex extraction and segmentation on developed turbulence
DE LA RECHERCHE À L’INDUSTRIE

Thank you

Nauleau Florent
Step: Persistence curve and diagram and groupdataset

Pipeline on the Kelvin Helmoltz Instability
**Step: Wasserstein**

- P parameter = 2
- Minimal relative precision = 0.01
- Saddle-max pairs
- Minimal relative persistence = 0.01