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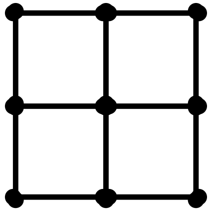
Topological Data Analysis for numerical method comparisons of 2D turbulent flows

HYPERION ingredients
HYPERsonic vehicle design with Immersed bOuNdaries

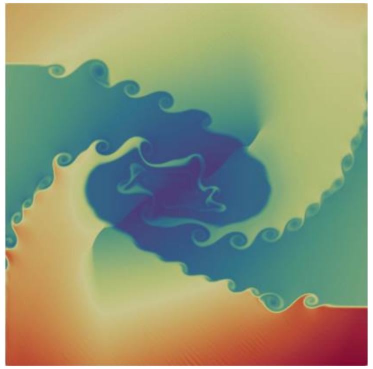
Finite Volume

$$\frac{d}{dt}\int_{\Omega}\mathbf{u}(\mathbf{x},t)d\mathbf{x} = \int_{\partial\Omega}\mathbf{F}(\mathbf{u}(\mathbf{x},t))\mathbf{n}ds$$

Cartesian Structured Mesh



Direct Numerical Simulation

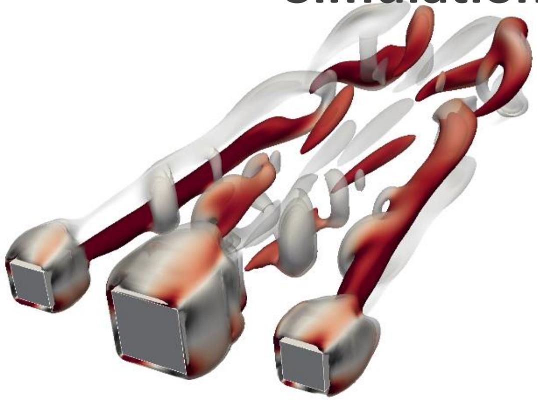


MPI

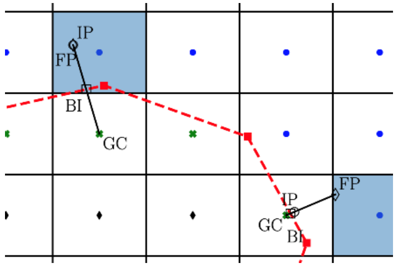
OpenMP

HYPERION

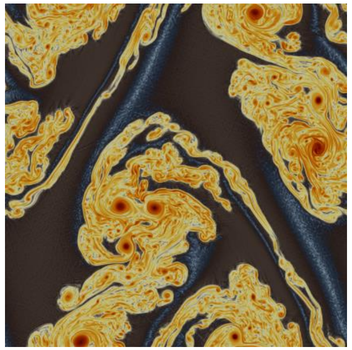
Large Eddy Simulation



Sharp Immersed Boundaries



Parallel I/O



VTK

ParaView

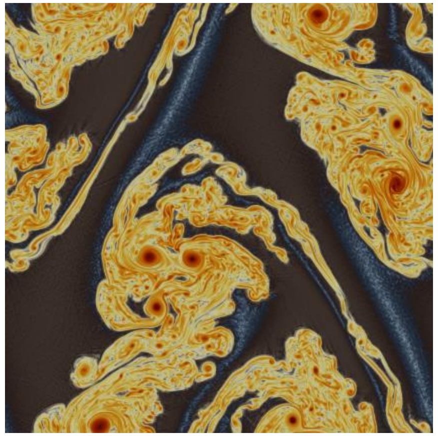
TTK

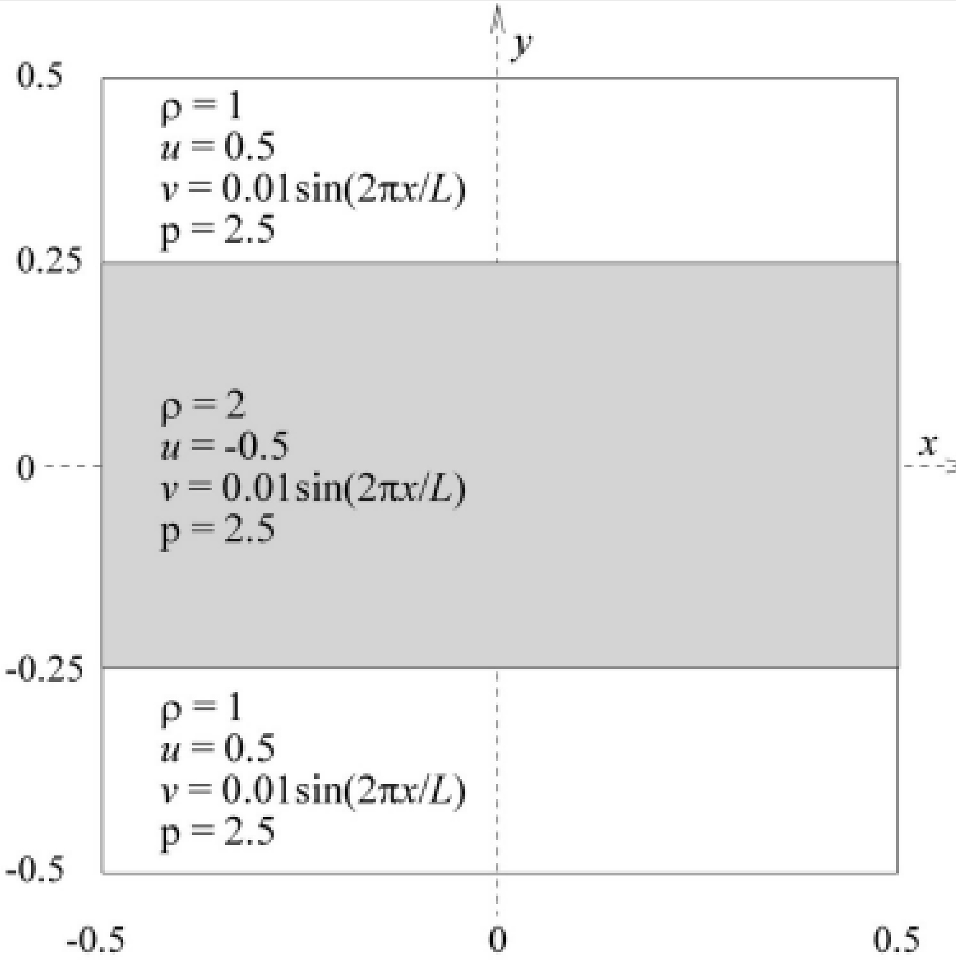
HDF

Turbulent flow : Kelvin-Helmholtz

Kelvin-Helmholtz Instability

- Pressure *(p)*, velocity *(u,v)* and density *(ρ)*
- Mesh size: 512*512 on Cartesian grid
- Boundary conditions: periodic

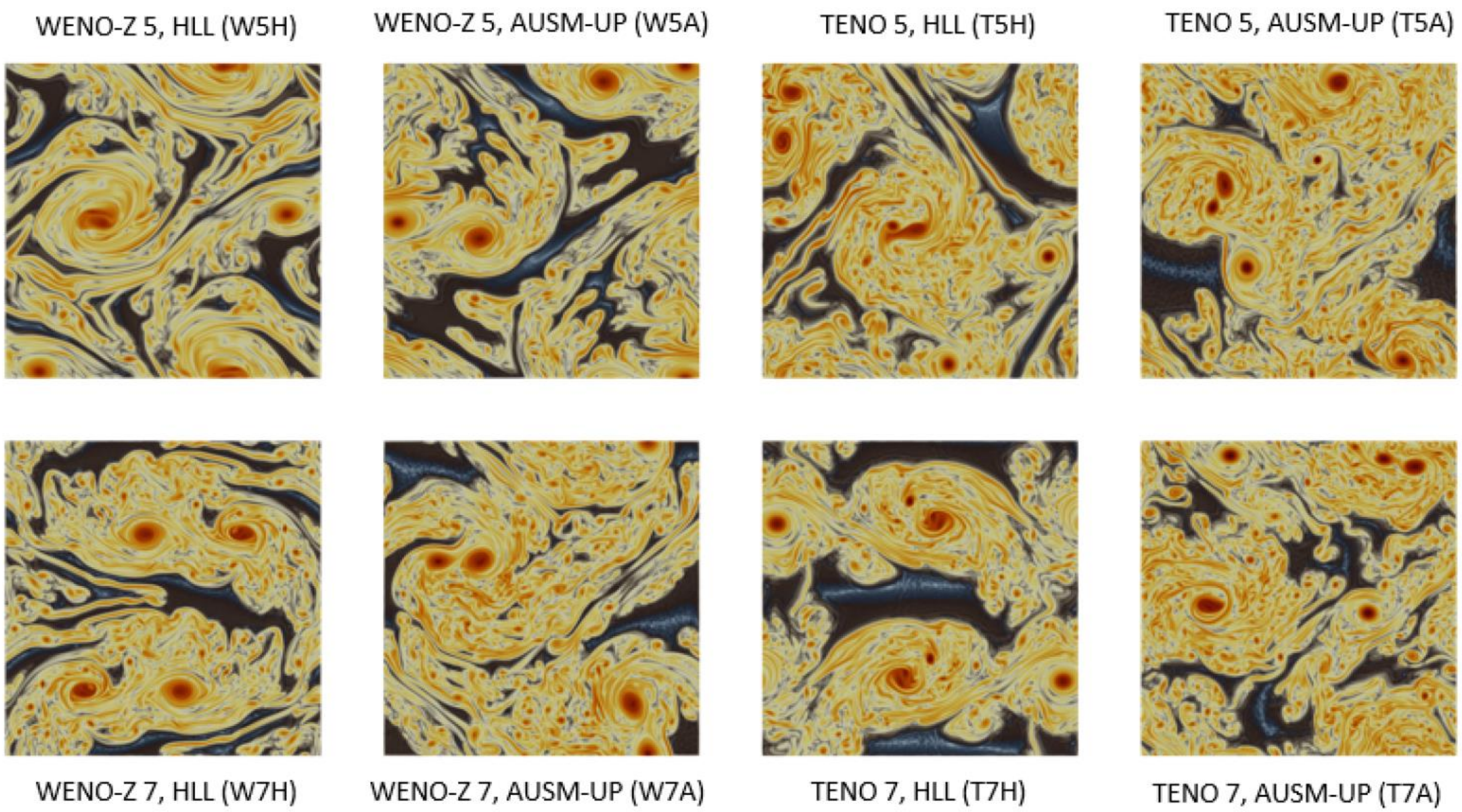




Dataset

- Input dataset : simplicial complexes, linear scalar field
- Common measure for turbulent flow, enstrophy: $\mathscr{E} = 0.5\,|\nabla \times \mathbf{u}|^2$

Find the best numerical method to reduce the global time of the simulation and help scientists to choose the best numerical method to describe 2D turbulent flow



Why to move to TDA ?

Complex and large dataset

- *Comparison between many different cases*
- *Comparison between different features: scales, vortices, recirculation bubble*
- *A lot of noise and perturbation with turbulent flow, need to use filter*

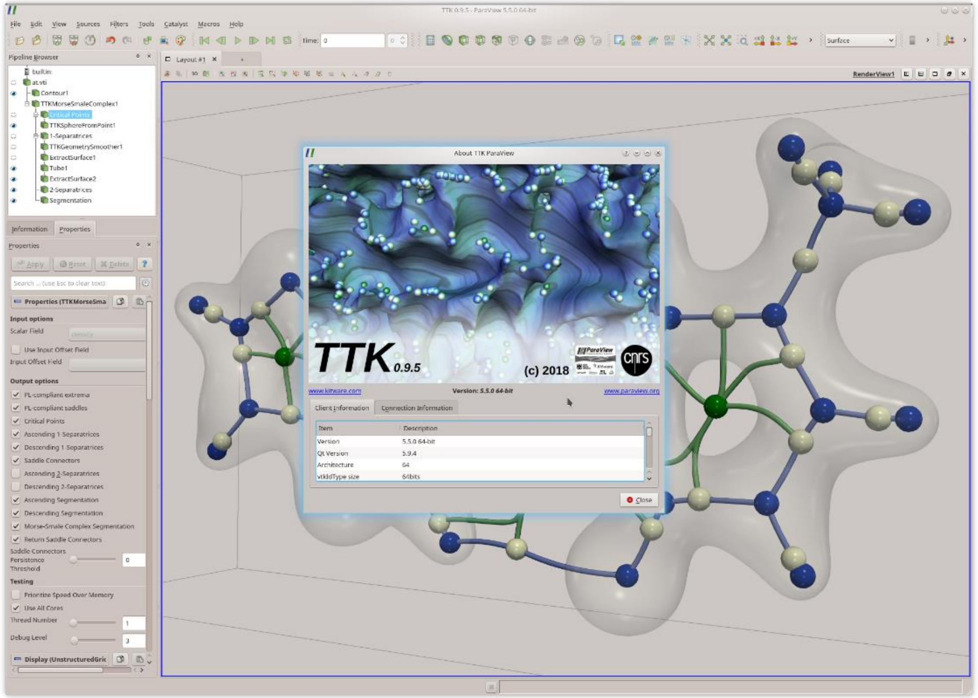
Why use topological data analysis

- *Identify vortex centers → Extract critical points*
- *Visual representation of the enstrophy maxima (critical points) → Persistent diagrams*
- *Noise removal of the enstrophy variable → Persistence threshold*
- *Comparison of all simulation runs → Wasserstein distance*

Topology ToolKit (TTK)

Open-source TDA library

- ~120k lines in C++, BSD license
- Python bindings, binary packages
- http://topology-tool-kit.github.io

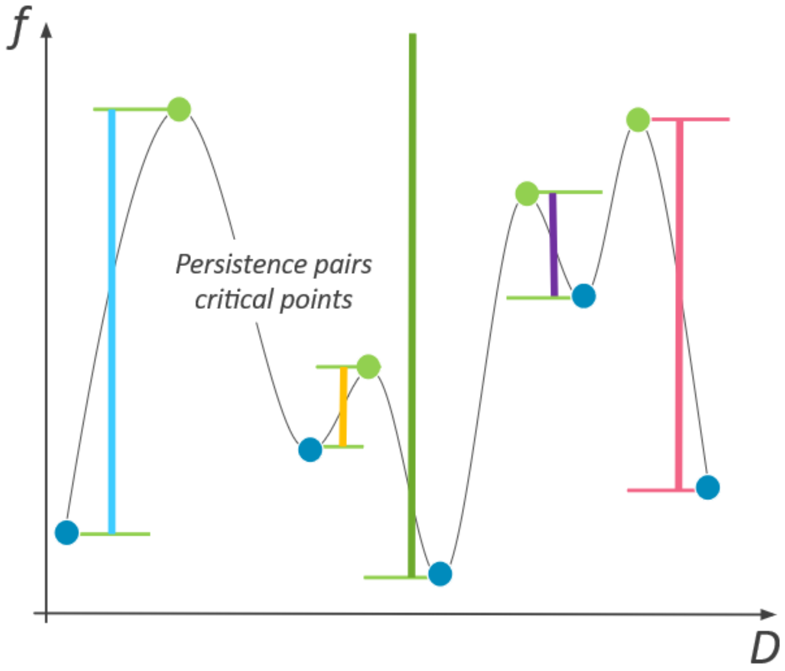


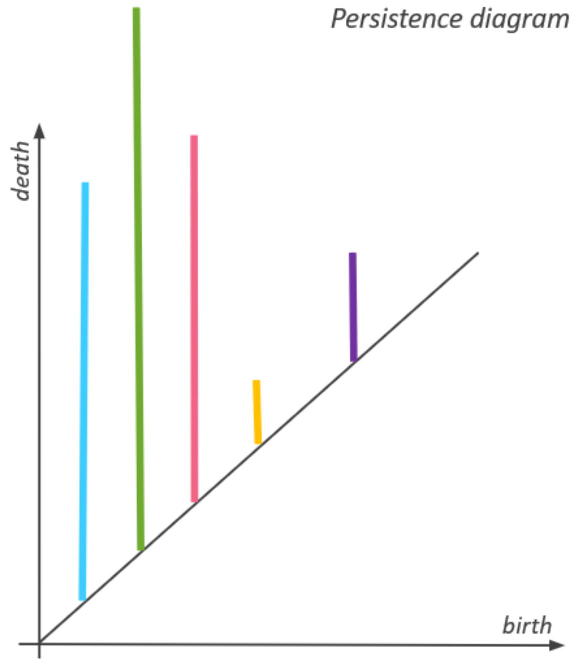
TTK provides

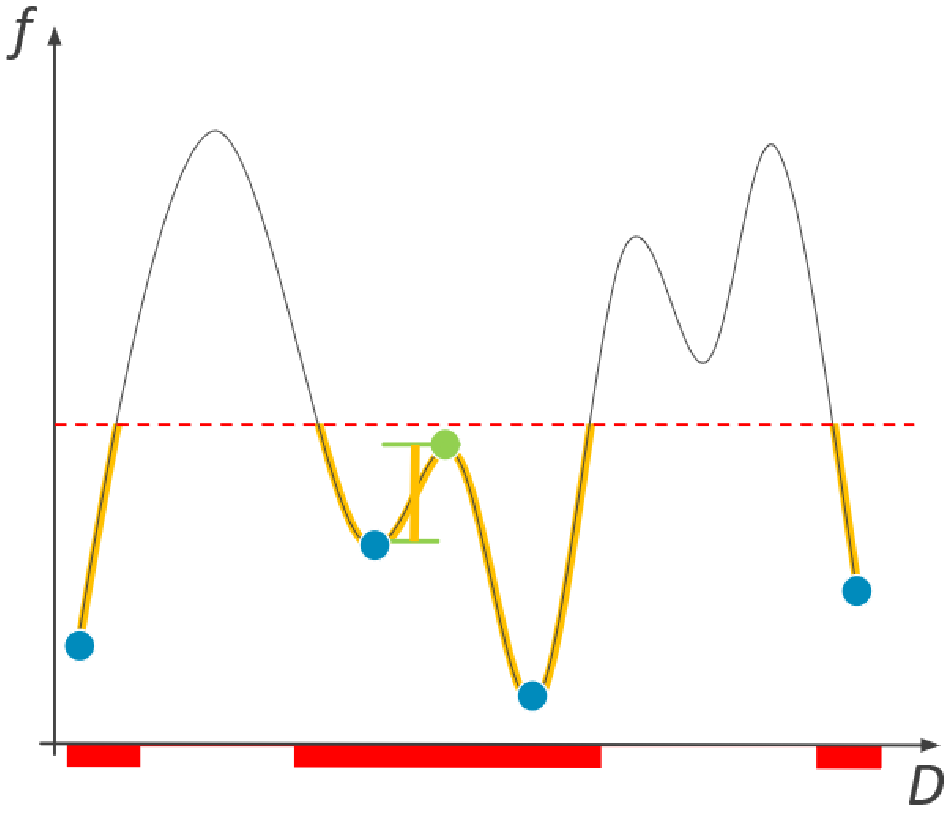
- Topological tools require to extract complex features
- End user analysis tool integrated into Paraview
- Great for interdisciplinary research !

Persistence

- Abstraction : Order topological features in term of importance or noise
- Evolution of the topology of sublevel sets
- Topological features are created (and destroyed) at critical points
- The lifetime of a topological feature is called "Persistence"

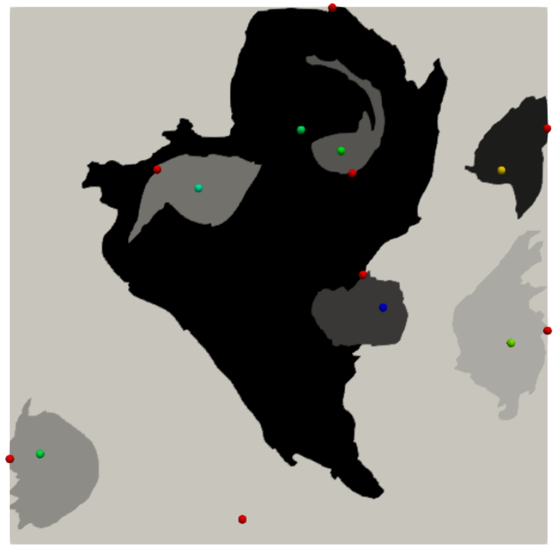


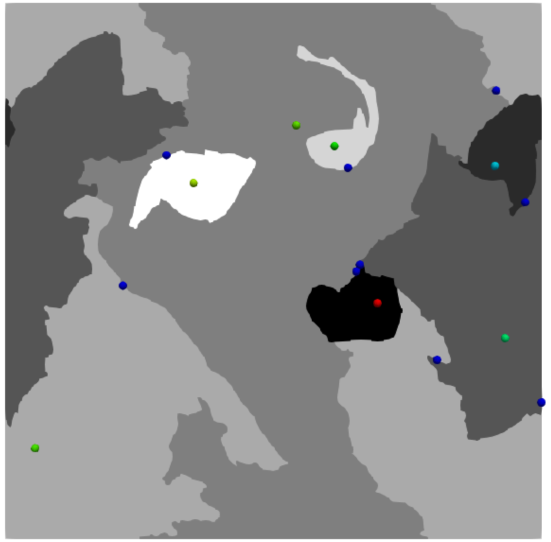




Periodic condition

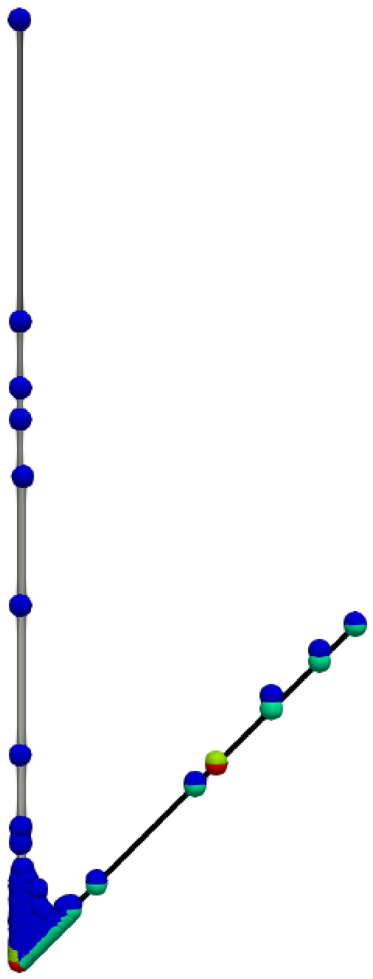
Non Periodic conditions



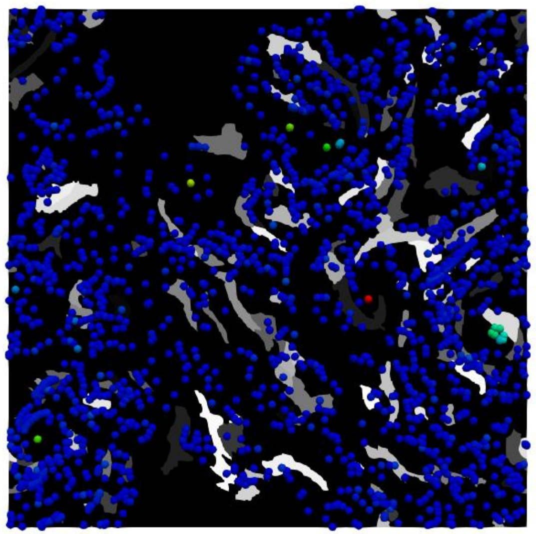


Periodic conditions

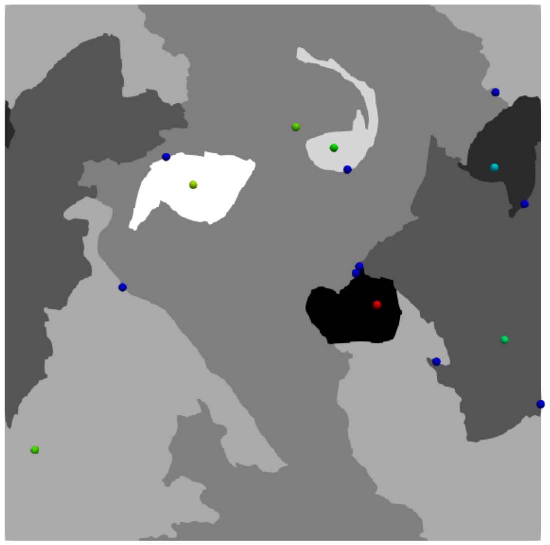
Use of the Persistence Homology : Filtering

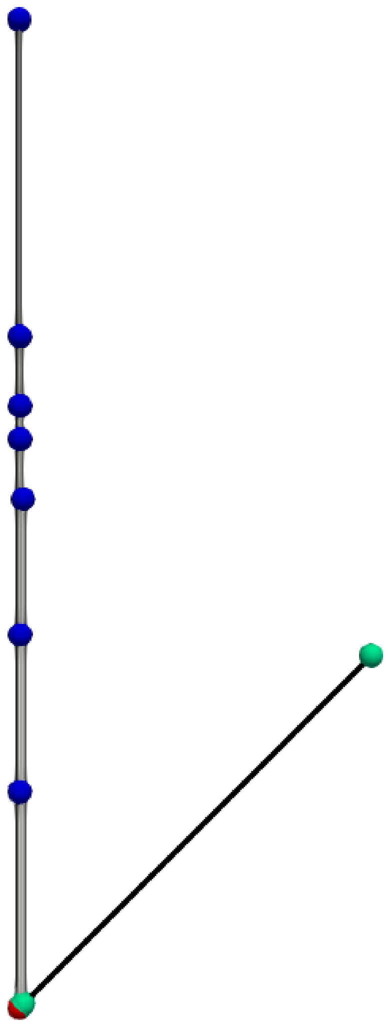


No Filter



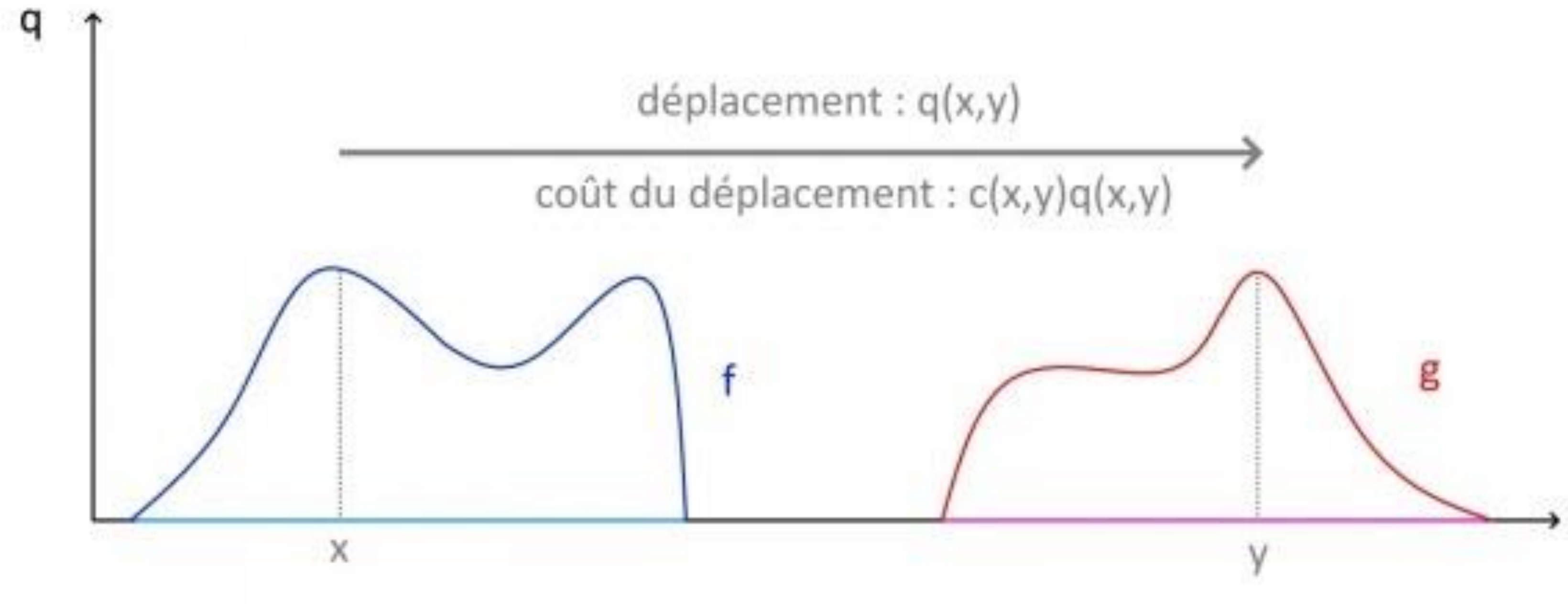
Filter



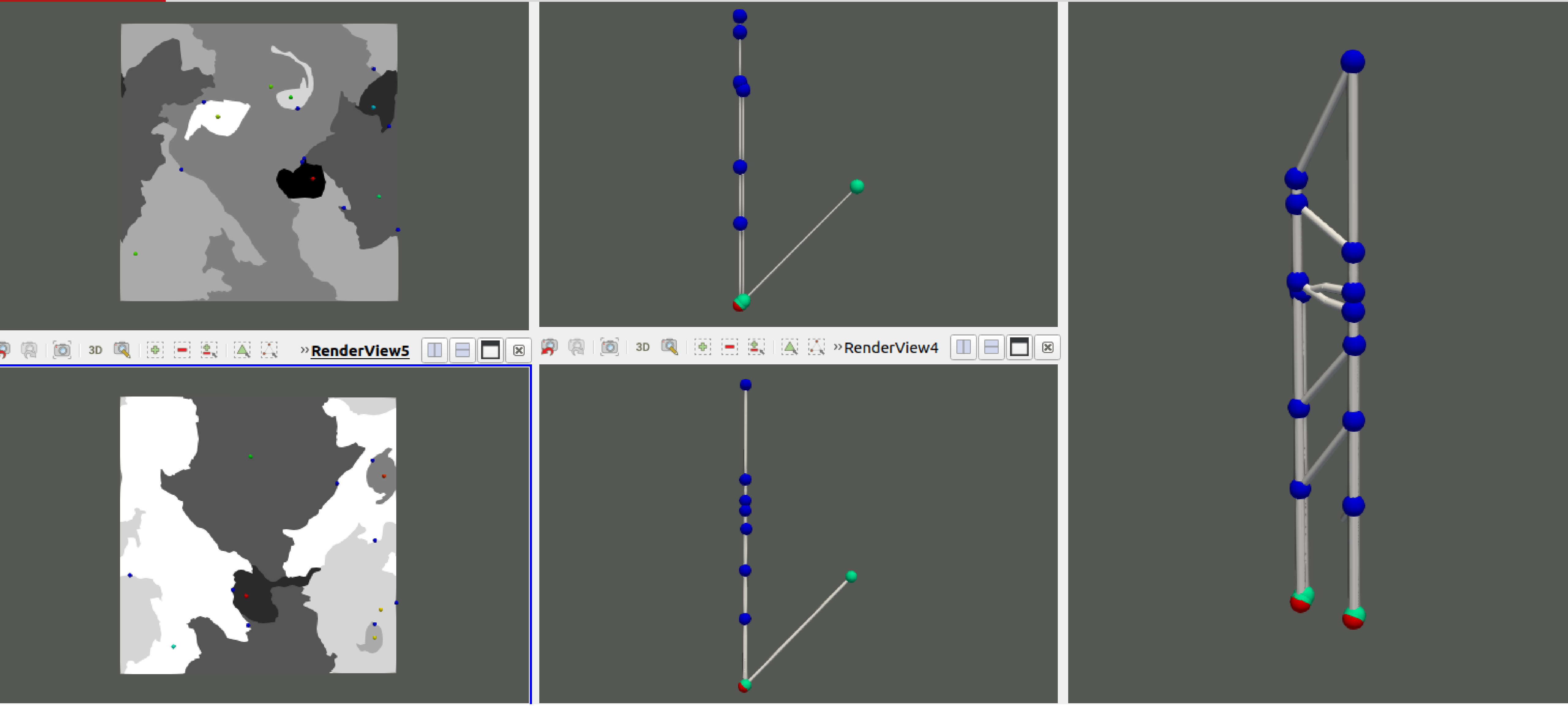


Wasserstein distance

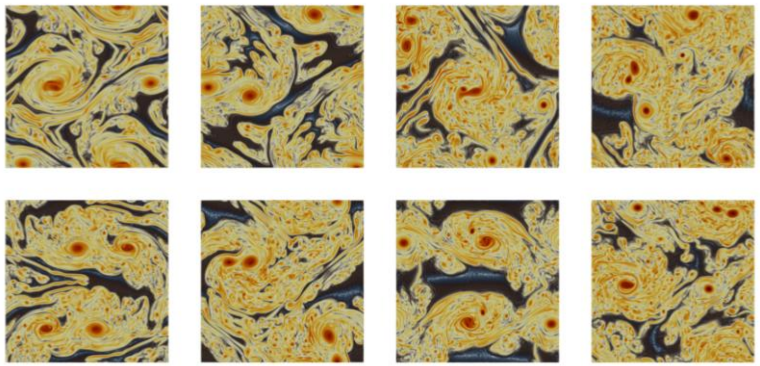
- Distance between distributions known as the « earth mover’s distance »
- Optimal transport problem : Minimal cost of moving one persistence diagram to the other



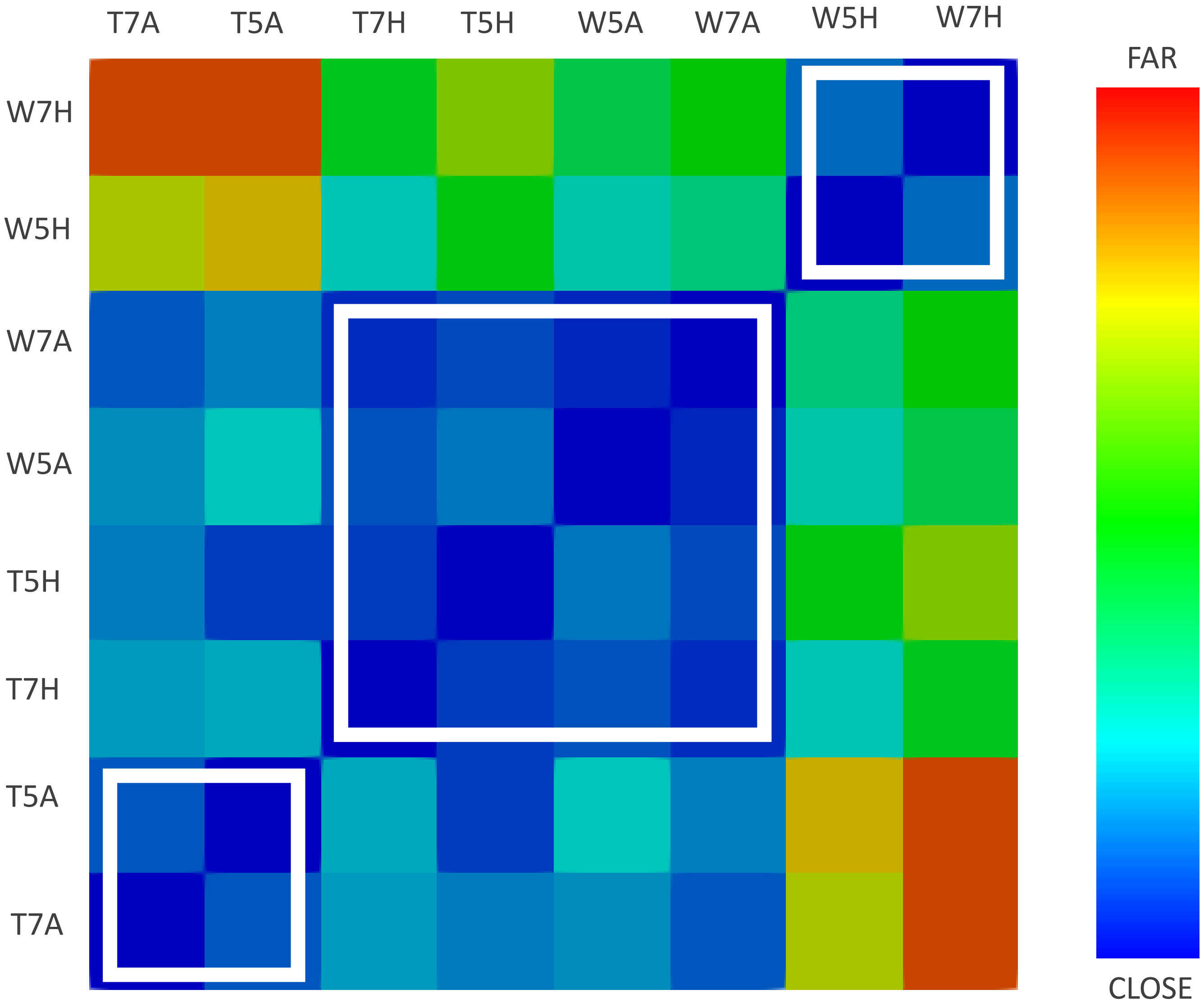
Use of the Persistence Homology : Wasserstein distance



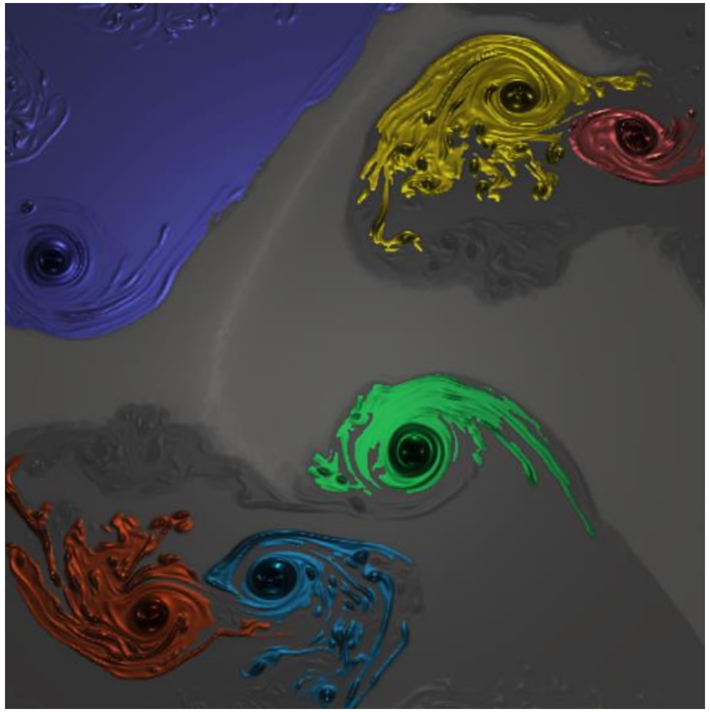
Matrix distance between numerical methods



WENOZ-5-HLL	W5H
WENOZ-5-AUSMUP	W5A
WENOZ-7-HLL	W7H
WENOZ-7-AUSMUP	W7A
TENO-5-HLL	T5H
TENO-5-AUSMUP	T5A
TENO-7-HLL	T7H
TENO-7-AUSMUP	T7A



Conclusion and Future work



Lessons learn thanks to TDA

- Confirm the independance of the orders
- Ease the identification of the scheme/order/solver

More to come …

- Apply this method at a larger scale (hundred of cases and runs)
  - More solvers
  - Different level of turbulence
  - Different mesh resolutions
- Continue with TDA for vortex extraction and segmentation on developed turbulence

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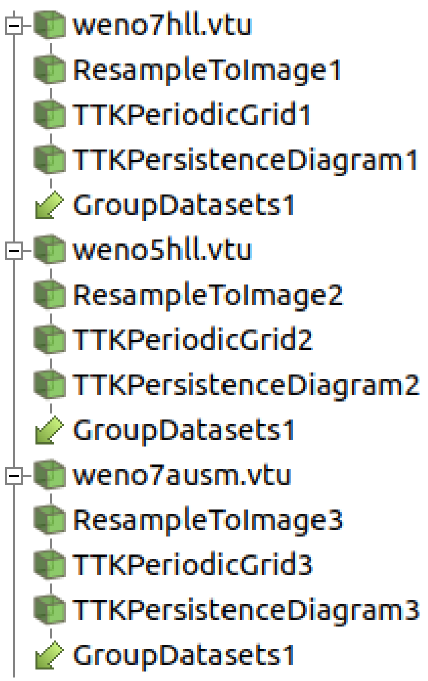
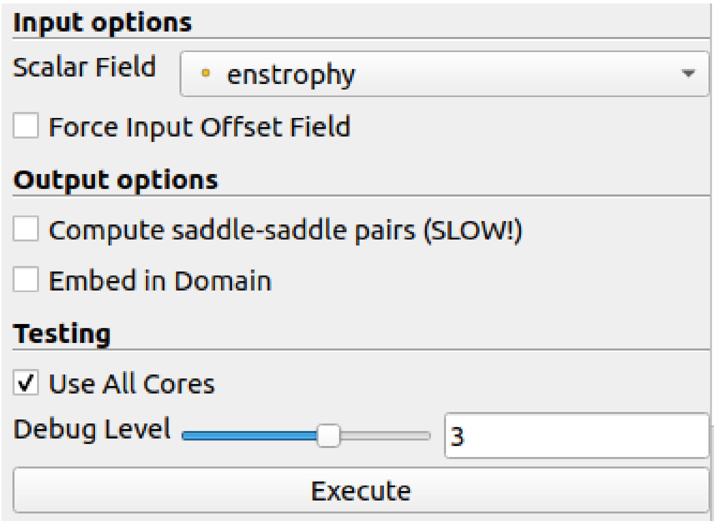
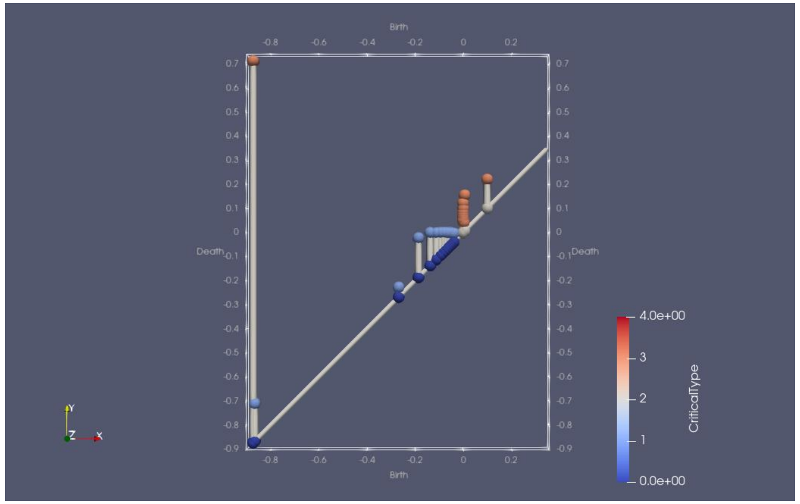
Thank you

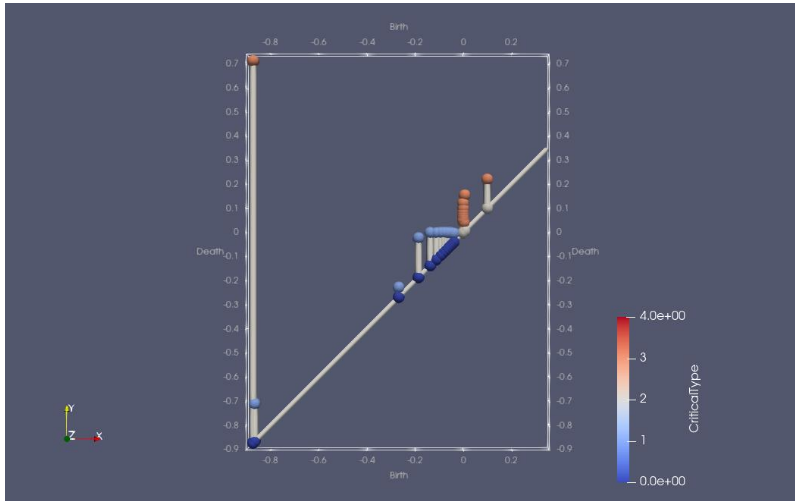
Nauleau Florent

Step : Persistence curve and diagram and groupdataset









Pipeline

Step : Wasserstein

- P parameter =2
- Minimal relative precision=0,01
- Saddle-max pairs
- Minimal relative persistence=0,01